Lecture 28

11.8 - Power Series

Def: A power series is a series of the form

Ex: $\sum_{n=0}^{\infty} \frac{x^n}{4^n}$. This series converges for some x and diverges for others. This is a geometric series with $r=\frac{x}{4}$, so as long as $\left|\frac{x}{4}\right| < 1 \Rightarrow |x| < 4$, it converges.

A power series defines a function

$$f(x) = c_0 + c_1 x + c_2 x^2 + \cdots = \sum_{n=0}^{\infty} c_n x^n$$

whose domain is the set of all for which the series converges.

Ex: Let $f(x) = \sum_{n=0}^{\infty} \frac{x^n}{4^n}$. What is the domain of f? What is f(0)?

Def: A **power series centered at a** is a power series of the form:

Ex: Around what value is the power series

$$\sum_{n=1}^{\infty} \frac{(5x-4)^n}{n^3}$$

centered? For what x does the series converge?

$\alpha\delta =$

Theorem: For any power series $\sum_{n=0}^{\infty} c_n (x-a)^n$, there are 3 possibilities for the values of x for which the series converges:

1) The series converges only for $x = a$
2) The series converges for all x
3) There is a number $R > 0$ such that the series converges if $|x-a| < R$ and diverges if $|x-a| > R$. The case(s) when $|x-a| = R$ must be checked individually.

Def: The <u>Radius of Convergence</u> of a power series is, as broken down in the 3 cases above:

1) 0 2) ∞ 3) R.

The <u>Interval of Convergence</u> is the interval of x values for which the series converges. In the 3 cases

1) $\{x\}$ 2) $(-\infty, \infty)$

3) can be any of:

$(a-R, a+R)$, $(a-R, a+R]$, $[a-R, a+R)$, $[a-R, a+R]$

Tip: It's usually best to use the ratio test to try to find the radius of convergence (and hence interval of convergence).

Ex: Find the radius of convergence and interval of convergence for the following power series:

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n}$ (c) $\displaystyle\sum_{n=1}^{\infty} n!(2x-1)^n$